NUCL 510 Take Home Quiz

1. One, Two, Three, Four, Five, and Six Group Problems

General form of n-group equation:

for n groups:

Separated Fluxes

Diffusion Coefficients:

Absorption Cross Sections:

Fission Cross Sections and Yield:

Fission Neutron Energy Probability:

Often we are given only or and , so those cases look like:

Scattering Cross Sections:

Because Self-Scattering cannot be defined as either loss or production, it can be considered zero and because we can assume no upscattering:

So, there will be general group equations (also using separation of variables in energy and space):

Dividing by

Converting to matrix form:

Where is a matrix. is given below

Now, There is only a solution when the the system is singular, so:

Which will give a system of equations, where k can be solved for.

Matlab was used to solve for these terms, which I’ve put in Appendix A to keep this methodology organized. The code was developed with the help of Neal Kostry and as such, our two codes will likely look similar.

The scattering source terms for two and three groups are shown below (with the “three-group addition” bolded).

Two-Group Scattering Source Term (group 1):

Three-Group Scattering Source Term (group 1):

This same change is seen in the fission term and the scattering loss term, as shown below.

Two-Group Scattering Loss Term (group 1):

Three-Group Scattering Loss Term (group 1):

Two-Group Fission Source Term (group 1):

Three-Group Fission Source Term (group 1):

As you can see, using a matrix to describe scattering and a vector to describe fission, the differences between group equations can be easily described using matrix algebra.

The general n-group addition is shown below, where is the desired number of groups, is the number of groups the addition will be added to (group 1):

1. Axial Flux Distribution for different criticalities

For critical case:

For Subproductive, Superproductive and Critical Case:

For Critically Productive case:

Trivial solution is given by methods above:

Now charting solutions found above:

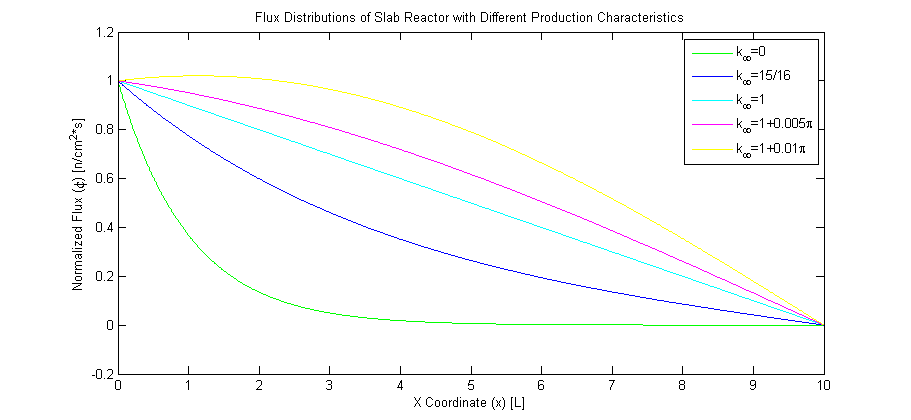


Figure 1 FLux Distributions of Slab Reactor with DIfferent Productions Characteristics

1. Two Group Spherical Core with Infinite Reflector

Solving Two Group Two Region Problem as laid out in (Lamarsh pp. 323-339):

Upon substituting values, the following fluxes can be obtained, with a chart (normalized to 1):

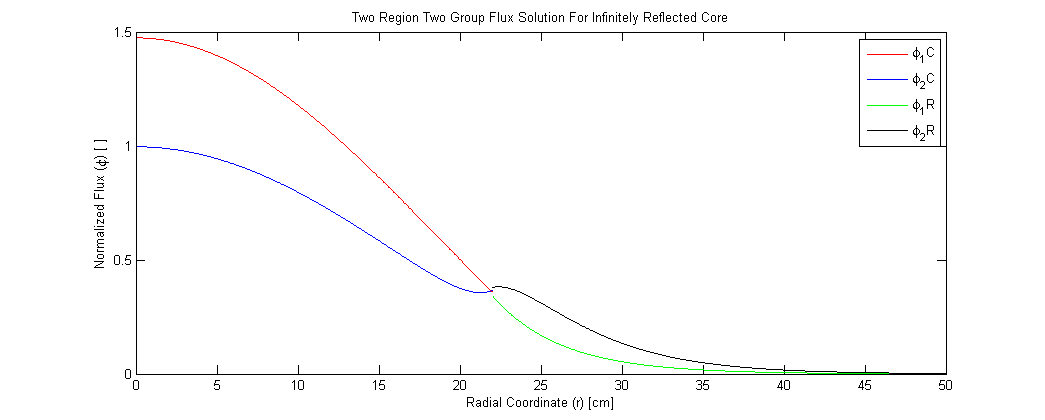


Figure 2 Two GroUp Two Region Problem

Now solving for criticality from (Lamarsh pp. 332-333):

Solving above determinant will give criticality of system.

# Appendix A – k Values Calculated with Matlab From Problem 1

The two-group, three-group, four-group, and four group (with two chi values) k values are shown below.

One-Group (:

kvalue =(chi\_1\*nuSigma\_f\_1)/(Sigma\_a\_1 + B\_squared\_1\*D\_1)

Two-Group (:

kvalue=(Sigma\_a\_2\*chi\_1\*nuSigma\_f\_1-Sigma\_s\_2to1\*chi\_1\*nuSigma\_f\_2+ B\_squared\_2\*D\_2\*chi\_1\*nuSigma\_f\_1)/(Sigma\_s\_2to1^2 + Sigma\_a\_1\*Sigma\_a\_2 + B\_squared\_1\*D\_1\*Sigma\_a\_2 + B\_squared\_2\*D\_2\*Sigma\_a\_1 + B\_squared\_1\*B\_squared\_2\*D\_1\*D\_2)

Three-Group (:

kvalue= (Sigma\_s\_3to2^2\*chi\_1\*nuSigma\_f\_1 + Sigma\_a\_2\*Sigma\_a\_3\*chi\_1\*nuSigma\_f\_1 - Sigma\_a\_3\*Sigma\_s\_2to1\*chi\_1\*nuSigma\_f\_2 - Sigma\_a\_2\*Sigma\_s\_3to1\*chi\_1\*nuSigma\_f\_3 + Sigma\_s\_2to1\*Sigma\_s\_3to2\*chi\_1\*nuSigma\_f\_3 - Sigma\_s\_3to1\*Sigma\_s\_3to2\*chi\_1\*nuSigma\_f\_2 + B\_squared\_2\*D\_2\*Sigma\_a\_3\*chi\_1\*nuSigma\_f\_1 + B\_squared\_3\*D\_3\*Sigma\_a\_2\*chi\_1\*nuSigma\_f\_1 - B\_squared\_3\*D\_3\*Sigma\_s\_2to1\*chi\_1\*nuSigma\_f\_2 - B\_squared\_2\*D\_2\*Sigma\_s\_3to1\*chi\_1\*nuSigma\_f\_3 + B\_squared\_2\*B\_squared\_3\*D\_2\*D\_3\*chi\_1\*nuSigma\_f\_1)/(Sigma\_a\_3\*Sigma\_s\_2to1^2 + Sigma\_a\_1\*Sigma\_s\_3to2^2 + Sigma\_a\_2\*Sigma\_s\_3to1^2 + Sigma\_a\_1\*Sigma\_a\_2\*Sigma\_a\_3 + B\_squared\_3\*D\_3\*Sigma\_s\_2to1^2 + B\_squared\_1\*D\_1\*Sigma\_s\_3to2^2 + B\_squared\_2\*D\_2\*Sigma\_s\_3to1^2 + B\_squared\_1\*D\_1\*Sigma\_a\_2\*Sigma\_a\_3 + B\_squared\_2\*D\_2\*Sigma\_a\_1\*Sigma\_a\_3 + B\_squared\_3\*D\_3\*Sigma\_a\_1\*Sigma\_a\_2 + B\_squared\_1\*B\_squared\_2\*D\_1\*D\_2\*Sigma\_a\_3 + B\_squared\_1\*B\_squared\_3\*D\_1\*D\_3\*Sigma\_a\_2 + B\_squared\_2\*B\_squared\_3\*D\_2\*D\_3\*Sigma\_a\_1 + B\_squared\_1\*B\_squared\_2\*B\_squared\_3\*D\_1\*D\_2\*D\_3)

Four-Group (:

kvalue = (Sigma\_a\_4\*Sigma\_s\_3to2^2\*chi\_1\*nuSigma\_f\_1 + Sigma\_a\_2\*Sigma\_s\_4to3^2\*chi\_1\*nuSigma\_f\_1 + Sigma\_a\_3\*Sigma\_s\_4to2^2\*chi\_1\*nuSigma\_f\_1 - Sigma\_s\_2to1\*Sigma\_s\_4to3^2\*chi\_1\*nuSigma\_f\_2 - Sigma\_s\_3to1\*Sigma\_s\_4to2^2\*chi\_1\*nuSigma\_f\_3 - Sigma\_s\_3to2^2\*Sigma\_s\_4to1\*chi\_1\*nuSigma\_f\_4 + Sigma\_a\_2\*Sigma\_a\_3\*Sigma\_a\_4\*chi\_1\*nuSigma\_f\_1 - Sigma\_a\_3\*Sigma\_a\_4\*Sigma\_s\_2to1\*chi\_1\*nuSigma\_f\_2 - Sigma\_a\_2\*Sigma\_a\_4\*Sigma\_s\_3to1\*chi\_1\*nuSigma\_f\_3 - Sigma\_a\_2\*Sigma\_a\_3\*Sigma\_s\_4to1\*chi\_1\*nuSigma\_f\_4 + Sigma\_a\_4\*Sigma\_s\_2to1\*Sigma\_s\_3to2\*chi\_1\*nuSigma\_f\_3 - Sigma\_a\_4\*Sigma\_s\_3to1\*Sigma\_s\_3to2\*chi\_1\*nuSigma\_f\_2 + Sigma\_a\_3\*Sigma\_s\_2to1\*Sigma\_s\_4to2\*chi\_1\*nuSigma\_f\_4 + Sigma\_a\_2\*Sigma\_s\_3to1\*Sigma\_s\_4to3\*chi\_1\*nuSigma\_f\_4 - Sigma\_a\_3\*Sigma\_s\_4to1\*Sigma\_s\_4to2\*chi\_1\*nuSigma\_f\_2 - Sigma\_a\_2\*Sigma\_s\_4to1\*Sigma\_s\_4to3\*chi\_1\*nuSigma\_f\_3 - Sigma\_s\_2to1\*Sigma\_s\_3to2\*Sigma\_s\_4to3\*chi\_1\*nuSigma\_f\_4 + Sigma\_s\_2to1\*Sigma\_s\_4to2\*Sigma\_s\_4to3\*chi\_1\*nuSigma\_f\_3 + Sigma\_s\_3to1\*Sigma\_s\_3to2\*Sigma\_s\_4to2\*chi\_1\*nuSigma\_f\_4 + Sigma\_s\_3to1\*Sigma\_s\_4to2\*Sigma\_s\_4to3\*chi\_1\*nuSigma\_f\_2 + Sigma\_s\_3to2\*Sigma\_s\_4to1\*Sigma\_s\_4to2\*chi\_1\*nuSigma\_f\_3 - Sigma\_s\_3to2\*Sigma\_s\_4to1\*Sigma\_s\_4to3\*chi\_1\*nuSigma\_f\_2 + B\_squared\_4\*D\_4\*Sigma\_s\_3to2^2\*chi\_1\*nuSigma\_f\_1 + B\_squared\_2\*D\_2\*Sigma\_s\_4to3^2\*chi\_1\*nuSigma\_f\_1 + B\_squared\_3\*D\_3\*Sigma\_s\_4to2^2\*chi\_1\*nuSigma\_f\_1 + B\_squared\_2\*D\_2\*Sigma\_a\_3\*Sigma\_a\_4\*chi\_1\*nuSigma\_f\_1 + B\_squared\_3\*D\_3\*Sigma\_a\_2\*Sigma\_a\_4\*chi\_1\*nuSigma\_f\_1 + B\_squared\_4\*D\_4\*Sigma\_a\_2\*Sigma\_a\_3\*chi\_1\*nuSigma\_f\_1 - B\_squared\_3\*D\_3\*Sigma\_a\_4\*Sigma\_s\_2to1\*chi\_1\*nuSigma\_f\_2 - B\_squared\_4\*D\_4\*Sigma\_a\_3\*Sigma\_s\_2to1\*chi\_1\*nuSigma\_f\_2 - B\_squared\_2\*D\_2\*Sigma\_a\_4\*Sigma\_s\_3to1\*chi\_1\*nuSigma\_f\_3 - B\_squared\_4\*D\_4\*Sigma\_a\_2\*Sigma\_s\_3to1\*chi\_1\*nuSigma\_f\_3 - B\_squared\_2\*D\_2\*Sigma\_a\_3\*Sigma\_s\_4to1\*chi\_1\*nuSigma\_f\_4 - B\_squared\_3\*D\_3\*Sigma\_a\_2\*Sigma\_s\_4to1\*chi\_1\*nuSigma\_f\_4 + B\_squared\_4\*D\_4\*Sigma\_s\_2to1\*Sigma\_s\_3to2\*chi\_1\*nuSigma\_f\_3 + B\_squared\_3\*D\_3\*Sigma\_s\_2to1\*Sigma\_s\_4to2\*chi\_1\*nuSigma\_f\_4 - B\_squared\_4\*D\_4\*Sigma\_s\_3to1\*Sigma\_s\_3to2\*chi\_1\*nuSigma\_f\_2 + B\_squared\_2\*D\_2\*Sigma\_s\_3to1\*Sigma\_s\_4to3\*chi\_1\*nuSigma\_f\_4 - B\_squared\_2\*D\_2\*Sigma\_s\_4to1\*Sigma\_s\_4to3\*chi\_1\*nuSigma\_f\_3 - B\_squared\_3\*D\_3\*Sigma\_s\_4to1\*Sigma\_s\_4to2\*chi\_1\*nuSigma\_f\_2 + B\_squared\_2\*B\_squared\_3\*D\_2\*D\_3\*Sigma\_a\_4\*chi\_1\*nuSigma\_f\_1 + B\_squared\_2\*B\_squared\_4\*D\_2\*D\_4\*Sigma\_a\_3\*chi\_1\*nuSigma\_f\_1 + B\_squared\_3\*B\_squared\_4\*D\_3\*D\_4\*Sigma\_a\_2\*chi\_1\*nuSigma\_f\_1 - B\_squared\_3\*B\_squared\_4\*D\_3\*D\_4\*Sigma\_s\_2to1\*chi\_1\*nuSigma\_f\_2 - B\_squared\_2\*B\_squared\_4\*D\_2\*D\_4\*Sigma\_s\_3to1\*chi\_1\*nuSigma\_f\_3 - B\_squared\_2\*B\_squared\_3\*D\_2\*D\_3\*Sigma\_s\_4to1\*chi\_1\*nuSigma\_f\_4 + B\_squared\_2\*B\_squared\_3\*B\_squared\_4\*D\_2\*D\_3\*D\_4\*chi\_1\*nuSigma\_f\_1)/(Sigma\_s\_2to1^2\*Sigma\_s\_4to3^2 + Sigma\_s\_3to1^2\*Sigma\_s\_4to2^2 + Sigma\_s\_3to2^2\*Sigma\_s\_4to1^2 + Sigma\_a\_3\*Sigma\_a\_4\*Sigma\_s\_2to1^2 + Sigma\_a\_1\*Sigma\_a\_4\*Sigma\_s\_3to2^2 + Sigma\_a\_2\*Sigma\_a\_4\*Sigma\_s\_3to1^2 + Sigma\_a\_1\*Sigma\_a\_2\*Sigma\_s\_4to3^2 + Sigma\_a\_1\*Sigma\_a\_3\*Sigma\_s\_4to2^2 + Sigma\_a\_2\*Sigma\_a\_3\*Sigma\_s\_4to1^2 + Sigma\_a\_1\*Sigma\_a\_2\*Sigma\_a\_3\*Sigma\_a\_4 - 2\*Sigma\_s\_2to1\*Sigma\_s\_3to1\*Sigma\_s\_4to2\*Sigma\_s\_4to3 + 2\*Sigma\_s\_2to1\*Sigma\_s\_3to2\*Sigma\_s\_4to1\*Sigma\_s\_4to3 - 2\*Sigma\_s\_3to1\*Sigma\_s\_3to2\*Sigma\_s\_4to1\*Sigma\_s\_4to2 + B\_squared\_3\*D\_3\*Sigma\_a\_4\*Sigma\_s\_2to1^2 + B\_squared\_4\*D\_4\*Sigma\_a\_3\*Sigma\_s\_2to1^2 + B\_squared\_1\*D\_1\*Sigma\_a\_4\*Sigma\_s\_3to2^2 + B\_squared\_2\*D\_2\*Sigma\_a\_4\*Sigma\_s\_3to1^2 + B\_squared\_4\*D\_4\*Sigma\_a\_1\*Sigma\_s\_3to2^2 + B\_squared\_4\*D\_4\*Sigma\_a\_2\*Sigma\_s\_3to1^2 + B\_squared\_1\*D\_1\*Sigma\_a\_2\*Sigma\_s\_4to3^2 + B\_squared\_1\*D\_1\*Sigma\_a\_3\*Sigma\_s\_4to2^2 + B\_squared\_2\*D\_2\*Sigma\_a\_1\*Sigma\_s\_4to3^2 + B\_squared\_2\*D\_2\*Sigma\_a\_3\*Sigma\_s\_4to1^2 + B\_squared\_3\*D\_3\*Sigma\_a\_1\*Sigma\_s\_4to2^2 + B\_squared\_3\*D\_3\*Sigma\_a\_2\*Sigma\_s\_4to1^2 + B\_squared\_3\*B\_squared\_4\*D\_3\*D\_4\*Sigma\_s\_2to1^2 + B\_squared\_1\*B\_squared\_4\*D\_1\*D\_4\*Sigma\_s\_3to2^2 + B\_squared\_2\*B\_squared\_4\*D\_2\*D\_4\*Sigma\_s\_3to1^2 + B\_squared\_1\*B\_squared\_2\*D\_1\*D\_2\*Sigma\_s\_4to3^2 + B\_squared\_1\*B\_squared\_3\*D\_1\*D\_3\*Sigma\_s\_4to2^2 + B\_squared\_2\*B\_squared\_3\*D\_2\*D\_3\*Sigma\_s\_4to1^2 + B\_squared\_1\*D\_1\*Sigma\_a\_2\*Sigma\_a\_3\*Sigma\_a\_4 + B\_squared\_2\*D\_2\*Sigma\_a\_1\*Sigma\_a\_3\*Sigma\_a\_4 + B\_squared\_3\*D\_3\*Sigma\_a\_1\*Sigma\_a\_2\*Sigma\_a\_4 + B\_squared\_4\*D\_4\*Sigma\_a\_1\*Sigma\_a\_2\*Sigma\_a\_3 + B\_squared\_1\*B\_squared\_2\*D\_1\*D\_2\*Sigma\_a\_3\*Sigma\_a\_4 + B\_squared\_1\*B\_squared\_3\*D\_1\*D\_3\*Sigma\_a\_2\*Sigma\_a\_4 + B\_squared\_1\*B\_squared\_4\*D\_1\*D\_4\*Sigma\_a\_2\*Sigma\_a\_3 + B\_squared\_2\*B\_squared\_3\*D\_2\*D\_3\*Sigma\_a\_1\*Sigma\_a\_4 + B\_squared\_2\*B\_squared\_4\*D\_2\*D\_4\*Sigma\_a\_1\*Sigma\_a\_3 + B\_squared\_3\*B\_squared\_4\*D\_3\*D\_4\*Sigma\_a\_1\*Sigma\_a\_2 + B\_squared\_1\*B\_squared\_2\*B\_squared\_3\*D\_1\*D\_2\*D\_3\*Sigma\_a\_4 + B\_squared\_1\*B\_squared\_2\*B\_squared\_4\*D\_1\*D\_2\*D\_4\*Sigma\_a\_3 + B\_squared\_1\*B\_squared\_3\*B\_squared\_4\*D\_1\*D\_3\*D\_4\*Sigma\_a\_2 + B\_squared\_2\*B\_squared\_3\*B\_squared\_4\*D\_2\*D\_3\*D\_4\*Sigma\_a\_1 + B\_squared\_1\*B\_squared\_2\*B\_squared\_3\*B\_squared\_4\*D\_1\*D\_2\*D\_3\*D\_4)

Four-Group (:

kvalue= (Sigma\_a\_4\*Sigma\_s\_3to2^2\*chi\_1\*nuSigma\_f\_1 + Sigma\_a\_4\*Sigma\_s\_3to1^2\*chi\_2\*nuSigma\_f\_2 + Sigma\_a\_2\*Sigma\_s\_4to3^2\*chi\_1\*nuSigma\_f\_1 + Sigma\_a\_3\*Sigma\_s\_4to2^2\*chi\_1\*nuSigma\_f\_1 + Sigma\_a\_1\*Sigma\_s\_4to3^2\*chi\_2\*nuSigma\_f\_2 + Sigma\_a\_3\*Sigma\_s\_4to1^2\*chi\_2\*nuSigma\_f\_2 - Sigma\_s\_2to1\*Sigma\_s\_4to3^2\*chi\_1\*nuSigma\_f\_2 + Sigma\_s\_2to1\*Sigma\_s\_4to3^2\*chi\_2\*nuSigma\_f\_1 - Sigma\_s\_3to1\*Sigma\_s\_4to2^2\*chi\_1\*nuSigma\_f\_3 - Sigma\_s\_3to2\*Sigma\_s\_4to1^2\*chi\_2\*nuSigma\_f\_3 - Sigma\_s\_3to2^2\*Sigma\_s\_4to1\*chi\_1\*nuSigma\_f\_4 - Sigma\_s\_3to1^2\*Sigma\_s\_4to2\*chi\_2\*nuSigma\_f\_4 + Sigma\_a\_2\*Sigma\_a\_3\*Sigma\_a\_4\*chi\_1\*nuSigma\_f\_1 + Sigma\_a\_1\*Sigma\_a\_3\*Sigma\_a\_4\*chi\_2\*nuSigma\_f\_2 - Sigma\_a\_3\*Sigma\_a\_4\*Sigma\_s\_2to1\*chi\_1\*nuSigma\_f\_2 + Sigma\_a\_3\*Sigma\_a\_4\*Sigma\_s\_2to1\*chi\_2\*nuSigma\_f\_1 - Sigma\_a\_2\*Sigma\_a\_4\*Sigma\_s\_3to1\*chi\_1\*nuSigma\_f\_3 - 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B\_squared\_3\*D\_3\*Sigma\_s\_2to1\*Sigma\_s\_4to1\*chi\_2\*nuSigma\_f\_4 + B\_squared\_3\*D\_3\*Sigma\_s\_2to1\*Sigma\_s\_4to2\*chi\_1\*nuSigma\_f\_4 - B\_squared\_4\*D\_4\*Sigma\_s\_3to1\*Sigma\_s\_3to2\*chi\_1\*nuSigma\_f\_2 - B\_squared\_4\*D\_4\*Sigma\_s\_3to1\*Sigma\_s\_3to2\*chi\_2\*nuSigma\_f\_1 + B\_squared\_1\*D\_1\*Sigma\_s\_3to2\*Sigma\_s\_4to3\*chi\_2\*nuSigma\_f\_4 + B\_squared\_2\*D\_2\*Sigma\_s\_3to1\*Sigma\_s\_4to3\*chi\_1\*nuSigma\_f\_4 - B\_squared\_1\*D\_1\*Sigma\_s\_4to2\*Sigma\_s\_4to3\*chi\_2\*nuSigma\_f\_3 - B\_squared\_2\*D\_2\*Sigma\_s\_4to1\*Sigma\_s\_4to3\*chi\_1\*nuSigma\_f\_3 - B\_squared\_3\*D\_3\*Sigma\_s\_4to1\*Sigma\_s\_4to2\*chi\_1\*nuSigma\_f\_2 - B\_squared\_3\*D\_3\*Sigma\_s\_4to1\*Sigma\_s\_4to2\*chi\_2\*nuSigma\_f\_1 + B\_squared\_1\*B\_squared\_3\*D\_1\*D\_3\*Sigma\_a\_4\*chi\_2\*nuSigma\_f\_2 + B\_squared\_2\*B\_squared\_3\*D\_2\*D\_3\*Sigma\_a\_4\*chi\_1\*nuSigma\_f\_1 + B\_squared\_1\*B\_squared\_4\*D\_1\*D\_4\*Sigma\_a\_3\*chi\_2\*nuSigma\_f\_2 + B\_squared\_2\*B\_squared\_4\*D\_2\*D\_4\*Sigma\_a\_3\*chi\_1\*nuSigma\_f\_1 + B\_squared\_3\*B\_squared\_4\*D\_3\*D\_4\*Sigma\_a\_2\*chi\_1\*nuSigma\_f\_1 + B\_squared\_3\*B\_squared\_4\*D\_3\*D\_4\*Sigma\_a\_1\*chi\_2\*nuSigma\_f\_2 - B\_squared\_3\*B\_squared\_4\*D\_3\*D\_4\*Sigma\_s\_2to1\*chi\_1\*nuSigma\_f\_2 + B\_squared\_3\*B\_squared\_4\*D\_3\*D\_4\*Sigma\_s\_2to1\*chi\_2\*nuSigma\_f\_1 - B\_squared\_1\*B\_squared\_4\*D\_1\*D\_4\*Sigma\_s\_3to2\*chi\_2\*nuSigma\_f\_3 - B\_squared\_2\*B\_squared\_4\*D\_2\*D\_4\*Sigma\_s\_3to1\*chi\_1\*nuSigma\_f\_3 - B\_squared\_1\*B\_squared\_3\*D\_1\*D\_3\*Sigma\_s\_4to2\*chi\_2\*nuSigma\_f\_4 - B\_squared\_2\*B\_squared\_3\*D\_2\*D\_3\*Sigma\_s\_4to1\*chi\_1\*nuSigma\_f\_4 + B\_squared\_1\*B\_squared\_3\*B\_squared\_4\*D\_1\*D\_3\*D\_4\*chi\_2\*nuSigma\_f\_2 + B\_squared\_2\*B\_squared\_3\*B\_squared\_4\*D\_2\*D\_3\*D\_4\*chi\_1\*nuSigma\_f\_1)/(Sigma\_s\_2to1^2\*Sigma\_s\_4to3^2 + Sigma\_s\_3to1^2\*Sigma\_s\_4to2^2 + Sigma\_s\_3to2^2\*Sigma\_s\_4to1^2 + Sigma\_a\_3\*Sigma\_a\_4\*Sigma\_s\_2to1^2 + Sigma\_a\_1\*Sigma\_a\_4\*Sigma\_s\_3to2^2 + Sigma\_a\_2\*Sigma\_a\_4\*Sigma\_s\_3to1^2 + Sigma\_a\_1\*Sigma\_a\_2\*Sigma\_s\_4to3^2 + Sigma\_a\_1\*Sigma\_a\_3\*Sigma\_s\_4to2^2 + Sigma\_a\_2\*Sigma\_a\_3\*Sigma\_s\_4to1^2 + Sigma\_a\_1\*Sigma\_a\_2\*Sigma\_a\_3\*Sigma\_a\_4 - 2\*Sigma\_s\_2to1\*Sigma\_s\_3to1\*Sigma\_s\_4to2\*Sigma\_s\_4to3 + 2\*Sigma\_s\_2to1\*Sigma\_s\_3to2\*Sigma\_s\_4to1\*Sigma\_s\_4to3 - 2\*Sigma\_s\_3to1\*Sigma\_s\_3to2\*Sigma\_s\_4to1\*Sigma\_s\_4to2 + B\_squared\_3\*D\_3\*Sigma\_a\_4\*Sigma\_s\_2to1^2 + B\_squared\_4\*D\_4\*Sigma\_a\_3\*Sigma\_s\_2to1^2 + B\_squared\_1\*D\_1\*Sigma\_a\_4\*Sigma\_s\_3to2^2 + B\_squared\_2\*D\_2\*Sigma\_a\_4\*Sigma\_s\_3to1^2 + B\_squared\_4\*D\_4\*Sigma\_a\_1\*Sigma\_s\_3to2^2 + B\_squared\_4\*D\_4\*Sigma\_a\_2\*Sigma\_s\_3to1^2 + B\_squared\_1\*D\_1\*Sigma\_a\_2\*Sigma\_s\_4to3^2 + B\_squared\_1\*D\_1\*Sigma\_a\_3\*Sigma\_s\_4to2^2 + B\_squared\_2\*D\_2\*Sigma\_a\_1\*Sigma\_s\_4to3^2 + B\_squared\_2\*D\_2\*Sigma\_a\_3\*Sigma\_s\_4to1^2 + B\_squared\_3\*D\_3\*Sigma\_a\_1\*Sigma\_s\_4to2^2 + B\_squared\_3\*D\_3\*Sigma\_a\_2\*Sigma\_s\_4to1^2 + B\_squared\_3\*B\_squared\_4\*D\_3\*D\_4\*Sigma\_s\_2to1^2 + B\_squared\_1\*B\_squared\_4\*D\_1\*D\_4\*Sigma\_s\_3to2^2 + B\_squared\_2\*B\_squared\_4\*D\_2\*D\_4\*Sigma\_s\_3to1^2 + B\_squared\_1\*B\_squared\_2\*D\_1\*D\_2\*Sigma\_s\_4to3^2 + B\_squared\_1\*B\_squared\_3\*D\_1\*D\_3\*Sigma\_s\_4to2^2 + B\_squared\_2\*B\_squared\_3\*D\_2\*D\_3\*Sigma\_s\_4to1^2 + B\_squared\_1\*D\_1\*Sigma\_a\_2\*Sigma\_a\_3\*Sigma\_a\_4 + B\_squared\_2\*D\_2\*Sigma\_a\_1\*Sigma\_a\_3\*Sigma\_a\_4 + B\_squared\_3\*D\_3\*Sigma\_a\_1\*Sigma\_a\_2\*Sigma\_a\_4 + B\_squared\_4\*D\_4\*Sigma\_a\_1\*Sigma\_a\_2\*Sigma\_a\_3 + B\_squared\_1\*B\_squared\_2\*D\_1\*D\_2\*Sigma\_a\_3\*Sigma\_a\_4 + B\_squared\_1\*B\_squared\_3\*D\_1\*D\_3\*Sigma\_a\_2\*Sigma\_a\_4 + B\_squared\_1\*B\_squared\_4\*D\_1\*D\_4\*Sigma\_a\_2\*Sigma\_a\_3 + B\_squared\_2\*B\_squared\_3\*D\_2\*D\_3\*Sigma\_a\_1\*Sigma\_a\_4 + B\_squared\_2\*B\_squared\_4\*D\_2\*D\_4\*Sigma\_a\_1\*Sigma\_a\_3 + B\_squared\_3\*B\_squared\_4\*D\_3\*D\_4\*Sigma\_a\_1\*Sigma\_a\_2 + B\_squared\_1\*B\_squared\_2\*B\_squared\_3\*D\_1\*D\_2\*D\_3\*Sigma\_a\_4 + B\_squared\_1\*B\_squared\_2\*B\_squared\_4\*D\_1\*D\_2\*D\_4\*Sigma\_a\_3 + B\_squared\_1\*B\_squared\_3\*B\_squared\_4\*D\_1\*D\_3\*D\_4\*Sigma\_a\_2 + B\_squared\_2\*B\_squared\_3\*B\_squared\_4\*D\_2\*D\_3\*D\_4\*Sigma\_a\_1 + B\_squared\_1\*B\_squared\_2\*B\_squared\_3\*B\_squared\_4\*D\_1\*D\_2\*D\_3\*D\_4)

# Appendix B – Matlab Code to Calculate k Values from Appendix A

clc;

for n=1:4

phi=sym('varphi\_%d',[n,1]);

Phi=sym('Phi');

PHI=phi\*Phi;

D=sym('D\_%d',[1,n]);

B\_squared=sym('B\_squared\_%d',[1,n]);

Sigma\_a=sym('Sigma\_a\_%d',[1,n]);

nuSigma\_f=sym('nuSigma\_f\_%d',[1,n]);

chi=sym('chi\_%d',[1,n]);

chi(2:n)=0;

Sigma\_s=sym('Sigma\_s\_%dto%d',[n,n]);

Sigma\_s(1:n+1:end)=0;

Sigma\_s=Sigma\_s-triu(Sigma\_s);

DB\_squared=D.\*B\_squared;

DB\_matrix=diag(DB\_squared);

Sigma\_a\_matrix=diag(Sigma\_a);

k=sym('k');

fission\_matrix=(1/k).\*transpose(chi)\*nuSigma\_f;

A=DB\_matrix+Sigma\_a\_matrix+Sigma\_s-transpose(Sigma\_s)-fission\_matrix;

equationtosolve=det(A);

kvalue=solve(equationtosolve,k)

end

n=4;

phi=sym('varphi\_%d',[n,1]);

Phi=sym('Phi');

PHI=phi\*Phi;

D=sym('D\_%d',[1,n]);

B\_squared=sym('B\_squared\_%d',[1,n]);

Sigma\_a=sym('Sigma\_a\_%d',[1,n]);

nuSigma\_f=sym('nuSigma\_f\_%d',[1,n]);

chi=sym('chi\_%d',[1,n]);

chi(3:n)=0;

Sigma\_s=sym('Sigma\_s\_%dto%d',[n,n]);

Sigma\_s(1:n+1:end)=0;

Sigma\_s=Sigma\_s-triu(Sigma\_s);

DB\_squared=D.\*B\_squared;

DB\_matrix=diag(DB\_squared);

Sigma\_a\_matrix=diag(Sigma\_a);

k=sym('k');

fission\_matrix=(1/k).\*transpose(chi)\*nuSigma\_f;

A=DB\_matrix+Sigma\_a\_matrix+Sigma\_s-transpose(Sigma\_s)-fission\_matrix;

equationtosolve=det(A);

kvalue=solve(equationtosolve,k)